Unit 1
Counting numbers

1.1 Numbers

$\mathbb{N}$ is the set of natural numbers, also called counting numbers.
$\mathbb{N} = \{1,\ 2,\ 3,\ 4\ldots\}$. Whole numbers are $\{0,\ 1,\ 2,\ 3,\ 4\ldots\}$. Read

0 Zero
1 One
2 Two
3 Three
4 Four
5 Five
6 Six
7 Seven
8 Eight
9 Nine

10 Ten
11 Eleven
12 Twelve
13 Thirteen
14 Fourteen
15 Fifteen
16 Sixteen
17 Seventeen
18 Eighteen
19 Nineteen
20 Twenty
21 Twenty-one
30 Thirty
40 Forty
50 Fifty
60 Sixty
70 Seventy
80 Eighty
90 Ninety
100 A/one hundred
200 Two hundred
300 Three hundred
400 Four hundred
600 Six hundred
800 Eight hundred
900 Nine hundred

We can use commas to write large numbers…

1,000 A/one thousand
1,001 A/one thousand and one
2,000 Two thousand
10,000 Ten thousand
100,000 A/one hundred thousand
1,000,000 A/one million

1.2 Ordinal numbers

An ordinal number describes the numerical position of an object.

1st First
2nd Second
3rd Third
4th Fourth
5th Fifth
6th Sixth
7th Seventh
8th Eighth
9th Ninth
10th Tenth
11th Eleventh
12th Twelfth
13th Thirteenth
14th Fourteenth
15th Fifteenth
16th Sixteenth
17th Seventeenth
18th Eighteenth
19th Nineteenth
20th Twentieth
21st Twenty-first
22nd Twenty-second
23rd Twenty-third
24th Twenty-fourth
25th Twenty-fifth
30th Thirtieth
31st Thirty-first
41st Forty-first
52nd Fifty-second
63rd Sixty-third
70th Seventieth
80th Eightieth
90th Ninetieth
91st Ninety-first
100th Hundredth

101st Hundred and first

1.3 Basic operations and expressions

1.3.1 Sum or addition:

$6 + 9 = 15$ six plus nine is equal to fifteen or six plus nine is fifteen .

1.3.2 Difference or subtraction:

$20 - 3 = 17$ twenty minus three is seventeen.

1.3.3 Order:

$2 < 8$ two is less than eight.
$12 > 5$ Twelve is greater than five.

1.3.4 Multiplication:

$5 \cdot 6 = 30$ five times six equals thirty or five multiplied by six is thirty.

1.3.5 Division:

$40 : 5 = 8$ forty divided by five is equal to eight: $\begin{array}{r} 8 \\ 5\overline{)40} \end{array}$

Sometimes a division is not exact, so you must write also the remainder.
Forty-two divided by five is eight and two left/remain because
$42 = 5 \cdot 8 + 2$.
Forty-two is the numerator, five is the denominator and two is the remainder.

$$\begin{array}{r} 8 \\ 5\ \overline{)42} \\ \underline{40} \\ 2 \end{array}$$

1.3.6 Bracket and square bracket:

To make groups we can use brackets ( ) and square brackets [ ].

1.4 Powers

Powers of numbers are made by repeated multiplication: a number multiplied by itself several times.
A power is made of two parts: the base is the number being multiplied, the index is the number of times you multiply.
Example: $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$, 3 is the base, 4 is the index or the exponent.
Read $3^4 = 81$ three to the fourth power is eighty-one or three to the power four is eighty-one.
When you square a number, you multiply it by itself.
Example: Read $5^2 = 25$ five squared is twenty-five or the square of five is twenty-five.
When you cube a number, you multiply it by itself three times.
Example: Read $5^3 = 125$ five cubed is a hundred and twenty-five or the cube of five is a hundred and twenty-five.

1.5 Order of Operations

1. Simplify the expressions inside grouping symbols, like brackets, square brackets, ...
2. Find the value of all powers and roots.

3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

1.6 Exercises

1 Write the correct number:
Forty-eight:
One hundred and two:
Two hundred and seventy-three:
Three thousand, nine hundred and eleven:
One hundred and two thousand, eight hundred and ten:
Two million, twenty thousand, five hundred and four:

2 How can we read these numbers?

63

245

504

1,617

3,024

10,981

123,450

3,413,012

3 Evaluate these arithmetic expressions and write down how we read the answer:
$3 \cdot 4 + 5 \cdot 2 =$

$13 + 5 \cdot 10 =$

$18 : 3 + 28 : 7 =$

$9 \cdot 8 - 54 : 6 =$

$20 \cdot 9 - 11 \cdot 7 + 81 : 9 =$

4 Put these numbers in order from least to greatest:
53025, 45422, 33452, 25242, 33542, 25232.

5 Do the sums and fill in the blank the right symbol (greater than or less than):

$12+54$ ______ $34+23$
$54+23$ ______ $49+25$
$12+7$ ______ $7+13$

$(34+40)+3$ ______ $16+60$
$13+12$ ______ $22+4$
$52+17$ ______ $28+45$

6 How can we read the following expressions?

$24 < 35$

$103 > 99$

$32+20=52$

$104-31=73$

$123 \cdot 100 = 12300$

$4234 : 2 = 2117$

7 Write how we read the following years:

1977
2001
1492

8 An athlete runs 1200m every day during a week. How many km does he run?

9 Ana has got €13; she wants to buy a toy, but she needs 21 euros more. How much does the toy cost?

10 Jaime has to read a book with 98 pages in a week. How many pages must he read each day?

11 Luis gets €6 every week and he spends 4 euros. How many weeks does he need to save 18 euros to buy a book?

12 A grandfather has €743 and three grandsons. He wants to give the same amount of money to each boy. How much does each boy get? Is there any money left?

13 Alfred has 55 Australian square coins. He wants to stick coins to get a big square. How many coins can he use to do it? How many coins left?

Unit 2
Divisibility

2.1 Divisibility

2.1.1 Factors

A factor/divisor of a number n is a number d which divides n.
d divides $n \Leftrightarrow d$ is a factor of n.
(read $\Leftrightarrow$ if and only if)
Example: $14 : 2 = 7$ so
two divides fourteen.

2.1.2 Divisible by

A number n is divisible by a number d if d divides n.
d divides $n \Leftrightarrow d$ is a factor of $n \Leftrightarrow n$ is divisible by d.

2.1.3 Multiples

A number n is a multiple of a number d if n is equal to d multiplied by another number.
d divides $n \Leftrightarrow d$ is a factor of $n \Leftrightarrow n$ is divisible by $d \Leftrightarrow n$ is a multiple of d.
Example: $14 : 2 = 7$ so
Two divides fourteen
Two is a factor of fourteen
Fourteen is divisible by two
Fourteen is a multiple of two.

2.2 Prime and composite numbers

2.2.1 Prime numbers

A prime number is a number that has only two factors 1 and the number itself.
1 is not considered a prime number as it only has one factor.
Example: 2 is a prime number because has only two factors: 1 and 2.

2.2.2 Composite numbers

A number with more than two factors it is called composite number.
Example: 6 is a composite number because has four factors:1, 2, 3 and 6.

2.2.3 Prime decomposition

To factorize a number you have to express the number as a product of its prime factors. Factorize a number by finding its prime decomposition. Prime decomposition is to find the set of prime factors of a number.

Example: $90 = 2 \cdot 45 = 2 \cdot 3 \cdot 15 = 2 \cdot 3 \cdot 3 \cdot 5$
We can simplify the product using powers properties $90 = 2 \cdot 3^2 \cdot 5$.

2.3 GCD and LCM

The Greatest Common Divisor (GCD) is the greatest number that is a common factor of two or more numbers. Example: $GCD(4, 14) = 2$, because factors of 4 are $1,\ 2,\ 4$, and factors of 14 are $1,\ 2,\ 7,\ 14$.
The Least Common Multiple (LCM) is the lowest number that is a common multiple of two or more numbers. Example: $LCM(6, 9) = 18$, because multiples of 6 are $6, 12, \overline{18}, 24\ldots$ and multiples of 9 are $9, \overline{18}, 27\ldots$

2.4 Exercises

1 Find every prime number and prime decomposition of each composite number.

	Prime	Composite	Neither	Prime decomposition
1				
2				
4				
17				
25				
91				
97				
121				
131				
193				

2 Find the GCF of these numbers using prime decompositions (write the prime factorization of each number and multiply only common factors)

12 and 20:

22 and 45:

14, 21 and 70:

30 and 42:

28 and 98:

121 and 99:

3 Find the LCM of these numbers (write the prime factorization of each number, identify all common prime factors and multiply them, find the product of the prime factors multiplying each common prime factor only once and any remaining factors).

2 and 15:

12 and 20:

8 and 12:

3, 6 and 15:

9 and 33:

13 and 39:

4 Anna sells bags of different kinds of cookies. She earns €6 selling bags of butter cookies, €12 selling chocolate cookies, and €15 selling bags of honey cookies. Each bag of cookies costs the same amount. What is the most that Anna could charge for each bag of cookies?

5 You have 60 pencils, 90 pens and 120 felt pens to make packages. Every pack has the same number of pencils, the same number of pens and the same number of felt pens.What is the maximum number of packages you can make using all of them? How many pencils has each package?

6 Fred, Eva, and Teresa each have the same amount of money. Fred has only 5 euro cents coins, Eva has only 20 euro cents coins, and Teresa has only 50 euro cents coins. What is the least amount of money that each of them could have?

7 Mandy, Lucy, and Danny each have bags of candy that have the same total weight. Mandy's bag has candy bars that each weigh 4 ounces, Lucy's bag has candy bars that each weigh 6 ounces, and Danny's bag has candy bars that each weigh 9 ounces. What is the least total weight that each of them could have?

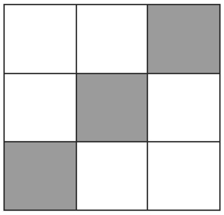
8 To write music in the stave you must divide beats by 2, 4, 8 or 16. What is the least amount of time you can write on a stave? What is the greatest divisor of a beat you need?

Unit 3
Fractions

3.1 Fraction

3.1.1 Definition

A fraction is a number that represents a part of something. Fractions are written in the form $\frac{a}{b}$, where a and b are naturals, and the number b is not zero. Read $\frac{a}{b}$ a over b.
The number a is called numerator, and the number b is called denominator.
Example:



$$\frac{3}{9}$$

3 is the numerator,
9 is the denominator.

3.1.2 Proper fractions and improper fractions

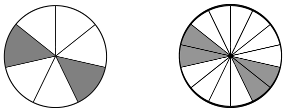
A proper fraction is a fraction such that $\frac{a}{b} < 1$. A fraction $\frac{a}{b} > 1$ is called an improper fraction.
Examples: $\frac{2}{7}$ is a proper fraction. $\frac{9}{7}$ is an improper fraction.

3.2 Equivalent fractions

3.2.1 Definition

Equivalent fractions are different fractions which represent the same amount.
Example:

$\frac{2}{7} = \frac{4}{14}$ because they represent the same amount.



3.2.2 Amplify and reduce fractions

To amplify a fraction we must multiply the numerator and the denominator by any number.
To reduce a fraction we must divide the numerator and the denominator by any common factor.

Example: To amplify $\frac{2}{7}$ we can multiply the numerator and the denominator by 3 $\Longrightarrow \frac{6}{21}$.
To reduce $\frac{15}{45}$ we can divide the numerator and the denominator by 5 $\Longrightarrow \frac{3}{9}$.

3.2.3 Simplest form of a fraction

A lowest terms fraction is a fraction that can not be reduced anymore. If you reduce a fraction to the lowest terms fraction you find the simplest form of the fraction.
To reduce a fraction to the simplest form, we can use two methods:

- Divide the numerator and the denominator by any common factor and keep dividing until there are no common factors (only 1).
- Divide the numerator and the denominator by their Greatest Common Factor.

Example:
To reduce $\frac{18}{60}$ to the lowest terms fractions we can divide by 2 and keep dividing until there are no common factors (only 1) $\frac{18}{60} = \frac{9}{30} = \frac{3}{10}$, and $GCF(3, 10) = 1$.
To reduce $\frac{18}{60}$ to the lowest terms fractions we can divide 18 and 60 by $GCF(18, 60) = 6$, so $\frac{18}{60} = \frac{3}{10}$.

3.3 Operations

3.3.1 Add and subtract

To add or subtract fractions with the same denominator, we have to add the numerators and keep the same denominator.

Example: $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$.

To add or subtract fractions with different denominators, first we have to amplify these fractions changing the denominators to the same number (using the LCM or any multiple of the denominators).
Examples: $\frac{3}{15} + \frac{1}{6} = \frac{11}{30}$, because
$LCM(15, 6) = 30$ $\quad \frac{3}{15} = \frac{6}{30}$ $\quad \frac{1}{6} = \frac{5}{30}$ $\quad \frac{6}{30} + \frac{5}{30} = \frac{11}{30}$.

3.3.2 Multiply and divide

To multiply fractions, multiply the numerators and multiply the denominators.

Example: $\frac{2}{5} \cdot \frac{3}{4} = \frac{2 \cdot 3}{5 \cdot 4} = \frac{6}{20} = \frac{3}{10}$.

You can get the reciprocal of a fraction by switching its numerator and denominator.

Example: The reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$.

To divide by a fraction, multiply by its reciprocal.

Example: $\frac{1}{8} : \frac{4}{7} = \frac{1}{8} \cdot \frac{7}{4} = \frac{1 \cdot 7}{8 \cdot 4} = \frac{7}{32}$.

3.4 Exercises

1 Complete:

A half=
$\frac{2}{2} =$
A/one third=
$\frac{2}{3} =$
A quarter=
$\frac{2}{4} =$
Two fifths=

$\frac{4}{5} =$
An/one eighth=
$\frac{1}{6} =$
Three quarters=
$\frac{3}{7} =$
Forty over twenty-three=
$\frac{35}{75} =$

2 Calculate the following and write down how we read the answer:

$\frac{3}{5} + \frac{1}{5} =$

$\frac{2}{4} + \frac{5}{6} =$

$\frac{1}{3} \cdot \left(\frac{1}{5} + \frac{4}{15}\right) =$

$\frac{3}{20} - \frac{2}{4} \cdot \frac{1}{5} =$

$\left(2 - \frac{1}{5}\right) : \frac{3}{10} =$

$\left(\frac{1}{2} + \frac{1}{3}\right) \cdot \frac{3}{5} =$

3 Fill in the blank with a number so the fractions are equivalent:

$\frac{\_}{7} = \frac{16}{14}$ $\frac{1}{5} = \frac{\_\_}{25}$ $\frac{144}{60} = \frac{12}{\_\_}$

$\frac{\_\_}{27} = \frac{11}{9}$ $\frac{11}{22} = \frac{\_}{6}$ $\frac{2}{5} = \frac{22}{\_\_}$

4 Order these fractions from least to greatest: $\frac{4}{6}, \frac{3}{2}, \frac{9}{15}, \frac{7}{5}, \frac{1}{3}, \frac{13}{6}$.

5 Find the lowest terms fraction dividing the numerator and denominator by common factors until the only common factor is 1:

$\frac{14}{6} =$ $\frac{18}{45} =$ $\frac{60}{80} =$ $\frac{22}{44} =$

6 Find the lowest terms fraction using prime decomposition of the numerator and denominator:

$\frac{18}{30} =$ $\frac{63}{42} =$ $\frac{14}{21} =$ $\frac{22}{33} =$

7 Find the lowest terms fraction dividing the numerator and denominator by the GCF:

$\frac{10}{6} =$ $\frac{30}{60} =$ $\frac{18}{24} =$ $\frac{23}{46} =$

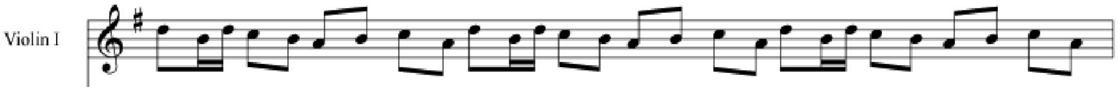
8 Alice runs $\frac{12}{5}$ miles on Monday. On Wednesday, she runs $\frac{11}{6}$ miles. How many miles does Alice run on both days?

9 Find:

(a) My soccer team wins 3 games and lost 5.
Write down a fraction of the games they win:

(b) Tim sells $\frac{3}{5}$ of his cookies, and now he has 14 left.
How many did he have originally?

(c) Of Tim's stone collection $\frac{1}{5}$ are white stones. He has total of 75 stones.
How many of them are not white?

10 When you write music in the stave you must write signs called notes, and you need to say how long a note is. The note value is a code which determines the note's duration. The length of a whole note 𝅝 is equal to four beats (in 4/4 time), half notes 𝅗𝅥 are played for one half the duration of the whole note, crotchet ♩ are played for one quarter the duration of the whole note, quavers ♪ are each played for one eighth the duration of the whole note, semiquaver 𝅘𝅥𝅯 are each played for one sixteenth the duration of the whole note,...
In this piece from Tchaikovsky's The Nutcracker draw a vertical line every two beats.

Unit 4
Decimals

4.1 Decimal expansion

The decimal expansion of a number is its representation in the decimal system.
Example: the decimal expansion of 25^2 is 625, of π is $3.14159\ldots$, and of $\frac{1}{9}$ is $0.1111\ldots$
Numbers can be placed to the left or right of a decimal point, to indicate values greater than one or less than one. The number to the left of the decimal point is a whole number.

4.2 Rational numbers and irrationals

The decimal expansion of a number may terminate, become periodic, or continue infinitely without repeating. All rational numbers have either finite decimal expansions (finite decimals) or repeating decimals. However, irrational numbers, neither terminate nor become periodic.

4.2.1 Finite decimal

A finite decimal is a positive number that has a finite decimal expansion.
Example: $1/2 = 0.5$ is a finite decimal.

4.2.2 Recurring decimal

A decimal number is a repeating/recurring decimal if at some point it becomes periodic: there is some finite sequence of digits that is repeated indefinitely. The repeating portion of a decimal expansion is conventionally denoted with a vinculum (a horizontal line placed above multiple quantities).
Example: $1/3 = 0.33333333\ldots = 0.\overline{3}$ is a recurring decimal.
Note that there are repeating decimals that begin with a non-repeating part.
Example: $1/30 = 0.03333333\ldots = 0.0\overline{3}$ is a recurring decimal that begin with a non-repeating part.

4.2.3 Irrationals

The decimal expansion of an irrational number never repeats or terminates.
Example: $\pi = 3.14159\ldots$ is an irrational.

4.3 Reading decimal numbers

When reading and writing decimals take note of the correct place of the last digit in the number. A decimal point means “and”. Remember that the value of a digit depends on its place

or position in the number. Look at the names of the different places of a figure:
Place(underlined) Name of position
$\underline{7}$,654,321.234567 Millions
7,$\underline{6}$54,321.234567 Hundred thousands
7,6$\underline{5}$4,321.234567 Ten thousands
7,65$\underline{4}$,321.234567 Thousands
7,654,$\underline{3}$21.234567 Hundreds
7,654,3$\underline{2}$1.234567 Tens
7,654,32$\underline{1}$.234567 Ones (units) position
7,654,321.$\underline{2}$34567 Tenths
7,654,321.2$\underline{3}$4567 Hundredths
7,654,321.23$\underline{4}$567 Thousandths
7,654,321.234$\underline{5}$67 Ten thousandths
7,654,321.2345$\underline{6}$7 Hundred Thousandths
7,654,321.23456$\underline{7}$ Millionths

Examples: Look at the following examples to learn how to read decimal numbers:
321.7 → Three hundred twenty-one and seven tenths
5,062.57 → Five thousand sixty-two and fifty-seven hundredths
43.27 → Forty-three point two seven
€4.67 → Four euros and sixty-seven cents
3.$\bar{4}$ → Three point four recurring

4.4 Operations with decimals

4.4.1 Adding and subtracting

Addition and subtraction of decimals is like adding and subtracting whole numbers. The only thing we must remember is to line up the place values correctly.

Examples:

To add $12.35 + 5.287$:
$$\begin{array}{cccccc} & 1 & 2 & .3 & 5 & \\ + & 5 & .2 & 8 & 7 & \\ \hline 1 & 7 & .6 & 3 & 7 & \end{array}$$

To subtract $12.993 - 2.28$:
$$\begin{array}{cccccc} & 1 & 2 & .9 & 9 & 3 \\ - & & 2 & .2 & 8 & \\ \hline & 1 & 0 & .7 & 1 & 3 \end{array}$$

4.4.2 Multiplying and dividing

When multiplying numbers with decimals, we first multiply them as if they were whole numbers. Then, the placement of the number of decimal places in the result is equal to the sum of the number of decimal places of the numbers being multiplied.

Example: To multiply 2.81 by 3.1:

$$\begin{array}{ccccc} & & 2 & .8 & 1 \\ \times & & & 3 & .1 \\ \hline & & 2 & 8 & 1 \\ & 8 & 4 & 3 & \\ \hline & 8 & .7 & 1 & 1 \end{array}$$

Division with decimals is easier to understand if the divisor is a whole number. In this case, when the decimal point appears in the dividend, we put it on the divisor.

Example: To divide 3.42 by 5:

$$\begin{array}{ccccc} 3 & 4. & 2 & /5 & \\ \cline{4-5} 0 & 4 & 2 & 6. & 8 \\ & & \underline{2} & & \end{array}$$

If the divisor has a decimal in it, we can make it a whole number by moving the decimal point the appropriate number of places to the right. If you move the decimal point to the right in the divisor, you must also do this for the dividend.
Example: To divide 13.34 by 3.2 we divide 133.4 by 32.

4.5 Approximating a quantity

Rounding off and truncating a decimal are techniques used to estimate or approximate a quantity. Instead of having a long string of figures, we can approximate the value of the decimal to a specified decimal place.

4.5.1 Truncating

To truncate a decimal, we leave our last decimal place as it is given and discard all digits to its right.
Example:
Truncate 123,235.23 to the tens place:123,230.
Truncate 123,235.23 to the tenth:123,235.2

4.5.2 Rounding off

After rounding off, the digit in the place we are rounding will either stay the same (referred to as rounding down) or increase by 1 (referred to as rounding up), then we discard all digits to its right.
To round off a decimal look at the digit to the right of the place being rounded:

- If the digit is 4 or less, the figure in the place we are rounding remains the same (rounding down).
- If the digit is 5 or greater, add 1 to the figure in the place we are rounding (rounding up).

- After rounding, discard all digits to the right of the place we are rounding.

Examples:
Round 123,235.23 to the tens place:123,240 we are rounding up.
Round 123,234.23 to the tens place:123,230 we are rounding down.

4.6 Exercises

1 We know that $234 \cdot 567 = 132{,}678$. Find $2.34 \cdot 5.67$:

2 Carmen earns €4.60 an hour working part-time as a private tutor. Last week she worked 6 hours. How much money did Carmen earn?

3 What is the cost of 3 pounds of jellybeans if each pound costs €2.30?

4 The length of a swimming pool is 16 feet. What is the length of the pool in yards? What is the length of the pool in meters? (Note 1 yard=3 feet=0.9144 meters).

5 The highest point in Alabama is Cheaha Mountain. It stands just a bit higher than 730 meters. What is this elevation in miles? (Note 1 km=5/8 miles)

6 Round 7.601 to the nearest whole number:
Truncate 68.94 to the tenth:
Round 68.94 to the nearest tenth:
Truncate 125.396 to the hundredth:
Round 125.396 to the nearest hundredth:

7 A can of beans costs €0.0726 per ounce. To the nearest cent, how much does an ounce of beans cost? how much does ten ounces of beans cost?

Unit 5
The metric system

5.1 The units of the metric system

The principal unit of length is the metre (m).
You can use submultiples like:
millimeter(1 mm= 0.00 m)
centimetre (1 cm= 0.01 m)
decimetre (1 dm= 0.1 m)
You can also use multiples like:
decameter (1 dam= 10 m)
hectometer (1 hm= 100 m)
kilometer (1 km= 1000 m).
The principal unit of capacity is the liter (l).
The principal unit of mass is the gram (g).
The area of a shape is a measure of the amount of space it covers. The principal unit of area is the square metre (m^2).
The volume of a 3D shape is a measure of the amount of space it occupies. The principal unit of volume is the cubic metre (m^3).
You can also use multiples and submultiples of those units.

5.2 Some other units

1 inch = 2.54 cm.
1 foot= 0.3048 m.
1 yard=3 feet= 0.9144 m.
1 mille = 1760 yards = 1.609344 km
1 gallon = 3.78 l.
1 pint = 0.473176 l.
1 ounce = 28.35 g.
1 acre = 4047 m^2.

Unit 6

Integers

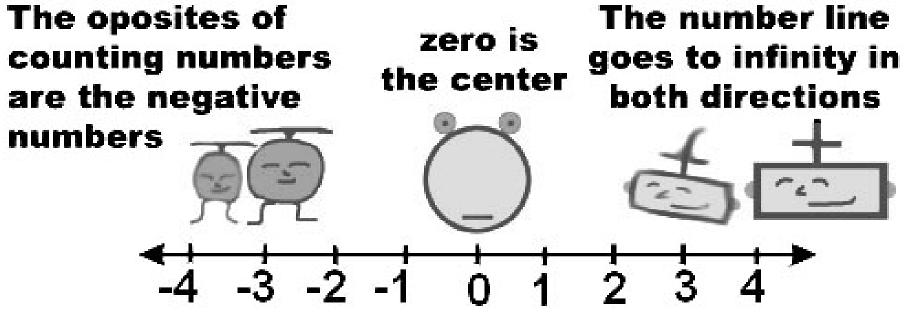
6.1 Sets of Numbers

$\mathbb{N}$ is the set of natural numbers, also called counting numbers or positive numbers. Positive numbers, negative numbers, and zero are called integers.
Positive numbers represent data that are greater than 0, they are written with a + sign or no sign at all.
Example: read $+3$ positive three or plus three.
Negative numbers represent data that are less than 0, they are written with a $-$ sign.
Example: read -5 negative five or minus five, for temperatures you can also use five below zero.

$\mathbb{Z}$ is the set of integers: $\mathbb{Z} = \{\ldots - 5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\ldots\}$
The absolute value of an integer is the value of the number regardless of its sign.
Example: read $|-2| = 2$ the absolute value of negative two is two.
The absolute value of a number is its distance from zero on a number line. Opposites are numbers that are the same distance from zero on a number line, but in opposite directions (so they both have the same absolute value).
Example: the opposite of -5 is $Op(-5) = 5$.

6.2 The number line

The number line is a straight line in which the integers are shown. The line continues left and right forever. If a number is to the left of a number on the number line, it is less than the other number. If it is to the right then it is greater than that number.

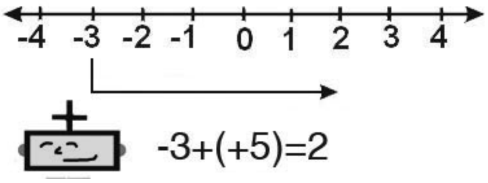


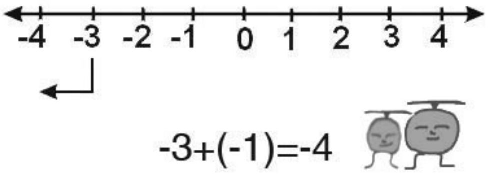
Example:
$2 < 4$ because 2 lies to the left of 4 in the number line.
$-1 > -3$ because -1 lies to the right of -3 in the number line.
$-4 < 1$ because -4 lies to the left of 1 in the number line.
$-2 < 0$ because 0 lies to the right of -2 in the number line.

6.3 Adding and subtracting integers

6.3.1 Add using the number line:

Add a positive integer by moving to the right on the number line.
Add a negative integer by moving to the left on the number line.





6.3.2 Add using absolute values:

If the signs are the same, add the numbers' absolute values and retain the same sign. The sum of two positive integers is always positive, the sum of two negative integers is always negative.

Examples:

Find $-3+(-2)$: the signs are the same (negative) and $|-3| = 3$, $|-2| = 2$, $3+2 = 5$, so $-3+(-2) = -5$.

Find $2+(+1)$: the signs are the same (positive) and $|+2| = 2$, $|+1| = 1$, $2+1 = 3$, so $2+(+1) = +3$.

If the signs are different, subtract the numbers' absolute values and retain the sign of the number with the greater absolute value.

Examples:

Find $-3+(+2)$: the signs are different and $|-3| = 3$, $|+2| = 2$, $3-2 = 1$, so $-3+(+2) = -1$, because $3 > 2$.

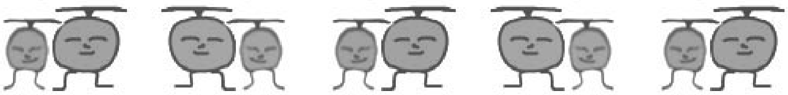
Find $-1+(+5)$: the signs are different and $|-1| = 1$, $|+5| = 5$, $5-1 = 4$, so $-1+(+5) = +4$, because $1 < 5$.

6.3.3 Subtraction:

Subtract an integer by adding its opposite.

Examples:

Find $-4-(-3)$: adding its opposite $-4-(-3) = -4+(+3) = -1$.
Find $-3-1$: adding its opposite $-3-1 = -3-(+1) = -3+(-1) = -4$.
Find $6-(-10)$: adding its opposite $6-(-10) = 6+(+10) = 16$.



6.4 Multiplying and dividing integers

6.4.1 Multiplying integers

To multiply integers, multiply the absolute values and then use these rules to find the corresponding sign:
The product of two integers with different signs is negative, the product of two integers with the same sign is positive.
Examples:
Find $2 \cdot (-1)$: the integers have different signs, the product is negative $2 \cdot (-1) = -2$.
Find $-4 \cdot 3$: the integers have different signs, the product is negative $-4 \cdot 3 = -12$.
Find $3 \cdot 5$: the integers have the same signs, the product is positive $3 \cdot 5 = +15$.
Find $-2 \cdot (-4)$: the integers have the same signs, the product is positive $-2 \cdot (-4) = +8$.

6.4.2 Dividing integers

To divide integers, divide the absolute values and then use these rules to find the corresponding sign:
The division of two integers with different signs is negative, the division of two integers with the same sign is positive.
Examples:
Find $5 : (-1)$: the integers have different signs, the quotient is negative $5 : (-1) = -5$.
Find $-12 : 3$: the integers have different signs, the quotient is negative $-12 : 3 = -4$.
Find $15 : 5$: the integers have the same signs, the quotient is positive $15 : 5 = +3$.
Find $-4 : (-2)$: the integers have the same signs, the quotient is positive $-4 : (-2) = +2$.

6.4.3 Rule of Signs for multiplying and dividing:

Unlike signs produce negative numbers:
$+ \cdot - = -, \quad - \cdot + = -. \qquad + : - = -, \qquad - : + = -.$
Like signs produce positive numbers:
$+ \cdot + = +, \quad - \cdot - = +. \qquad + : + = +, \qquad - : - = +.$

6.5 Exercises

1 Write the correct number:
Positive forty-eight:
Negative one hundred and two:
Negative two hundred and seventy-three:
Three thousand, nine hundred and eleven:

Negative one hundred and two thousand, eight hundred and ten:
Positive two million, twenty thousand, five hundred and fifty-four:

2 How can we read these numbers?

-34:
435:
-543:
-1,207:
5,673:
-12,856:
-174,730:
3,323,045:
1,100,305:

3 Complete:

- $+2$ is ____________ than 5, because $+2$ lies to the left of 5 in the number line.
- -5 is ____________ than -3, because -5 ______ to the ___________ of -3.
- -8 is ____________ than 0, because 0 lies ____ the ____________ of -8.
- 3 is ____________ than -2, because -2 _______ to the ____________ of 3.

4 Put these numbers in order from least to greatest:
$-25, 22, -3, 42, 31, 2$

5 Find out the opposite of each number. Write down a sentence and the expression:

-3

4

2

-12

6 Find out the absolute value of each number. Write down a sentence and the expression:

5

-7

-9

11

-23

0

7 Find:

$4 + (-1) =$
$16 - 11 =$
$12 - 15 =$
$7 - (-4) =$
$-8 + 2 =$
$(-20) + (-9) =$

8 Find:

$13 + (-1) - 14 - (-2) =$

$8 + (-15) - 13 + 24 =$

$2 - 8 + 2 - 5 + 2 =$

$1 + (-2) + 6 - 12 + 6 + (-6) =$

$-17 + 7 - 4 - 11 =$

9 Find:

$(-3) \cdot 4 - 5 \cdot 2 =$

$13 - 5 \cdot 10 =$

$8 : (-4) + 14 : 7 =$

$2 \cdot (-3) - 25 : (-5) =$

$-9 \cdot 2 - 54 : 6 =$

$(-20) \cdot (-9) - 11 \cdot 7 - 81 : 9 =$

10 Write an integer beside each sentence:
Mary hikes at a height of two thousand and twenty metres above sea level:
Luis earns €1070 a month:
An Alfa class submarine can operate at 1300 meters:
The plane flies three thousand metres high:
The temperature outside is five degrees Celsius bellow zero:

11 Thales of Miletus was born in 624 b.C, and he lived 78 years. Find out the year of his death.

12 Parts of Death Valley in California are below sea level. A hiker starts at an elevation of 12 feet below sea level. Then she hikes to an elevation that is 8 feet above sea level. How many feets does she hike?

13 Jackie buys three identical shirts in different colors. She has to pay €3.24 in taxes. The total amount she pays is €57.24. What is the cost of each shirt without the taxes?

14 Andrew and Jacob collect aluminum cans to recycle. Andrew has 56 cans. This is eighteen more cans than Jacob has. How many cans does Jacob have?

Unit 7
Algebraic expressions and equations

7.1 Monomials:

A variable is a symbol.
An algebraic expression in variables $x,\ y,\ z,\ a,\ r,\ t\ldots\ k$ is an expression constructed with the variables and numbers using addition, multiplication, and powers.
A number multiplied with a variable in an algebraic expression is named coefficient.
A product of positive integer powers of a fixed set of variables multiplied by some coefficient is called a monomial.
Examples: $3x$, $\frac{2}{3}xy^2$, x^2y^3z.
In a monomial with only one variable, the power is called its order, or sometimes its degree.
Example: Deg($5x^4$)=4.
In a monomial with several variables, the order/degree is the sum of the powers.
Example: Deg(x^2z^4)=6.
Monomials are called similar or like ones, if they are identical or differed only by coefficients.
Example: $2x^3y^2$ and $\frac{2}{5}x^3y^2$ are like monomials. $4xy^2$ and $4y^2x^4$ are unlike monomials.

7.2 Adding and subtracting monomials:

You can ONLY add/subtract like monomials. To add/subtract like monomials use the same rules as with integers.

Example: $3x + 4x = (3 + 4)x = 7x$.
Example: $20a - 24a = (20 - 24)a = -4a$.

7.3 Identities and Equations:

An equation is a mathematical expression stating that a pair of algebraic expression are the same. If the equation is true for every value of the variables then its called Identity. An identity is a mathematical relationship equating one quantity to another which may initially appear to be different.
Example: $x^2 - x^3 + x + 1 = 3x^4$ is an equation, $3x^2 - x + 1 = x^2 - x + 2 + 2x^2 - 1$ is an identity.
In an equation: the variables are named unknowns (or indeterminate quantities), the number multiplied with a variable is named coefficient, a term is a summand of the equation, the highest power of the unknowns is called the order/degree of the equation.
Example: In the equation $2x^3 + 4y + 1 = 4$ the unknowns are x and y, the coefficient of x^3 is 2 and the coefficient of y is 4, the order of the equation is 3.

7.4 Solving:

7.4.1 Solution:

You are solving a equation when you replace a variable with a value and the mathematical expressions are still the same. The value for the variables is the solution of the equation.
Example: Sam is 9 years old. This is seven years younger than her sister Rose's age. We can solve an equation to find Rose's age: $x - 7 = 9$, the solution of the equation is 16, so Rose is 16 years old.

7.4.2 The balance method:

To solve equations you can use the balance method, you must carry out the same operations in both sides and in the same order. You must use these properties:

- Addition Property of Equalities: If you add the same number to each side of an equation, the two sides remain equal (note you can also add negative numbers).
- Multiplication Property of Equalities: If you multiply by the same number each side of an equation, the two sides remain equal (note you can also multiply by fractions).
- Brackets: Sometimes you will need to solve equations involving brackets. If brackets appear, first remove the brackets by expanding each bracketed expression.

Example: Solve $4x + 3 \cdot (x - 25) = 240$:
First we remove brackets: $3 \cdot (x - 25) = 3x - 75$ so
$4x + 3x - 75 = 240$.
Them we use addition property:
$4x + 3x - 75 + 75 = 240 + 75 \Longrightarrow 4x + 3x = 240 + 75 \Longrightarrow 7x = 315$.
Now we can use multiplication property:

$$\frac{7}{7}x = \frac{315}{7}$$

so the solution is $x = 45$.

7.5 Exercises

1 Solve the equations:

(a) $4x + 2 = 26$

(b) $5(2x - 1) = 7(9 - x)$

(c) $\frac{x}{2} + \frac{2x}{3} = 7$

(d) $19 + 4x = 9 - x$

(e) $3(2x + 1) = x - 2$

(f) $\frac{x}{5} - \frac{3x}{10} = \frac{1}{5}$

2 Find a number such that 2 less than three times the number is 10.

3 Mr. Roberts and his wife have 370 pounds. Mrs. Roberts has 155 pounds less than twice her husband's money. How many pounds does Mr. Roberts have? How many pounds does Mrs. Roberts have?

4 The length of a room exceeds the width by 5 feet. The length of the four walls is 30 feet. Find the dimensions of the room.

5 Maria spent a third of her money on food. Then, she spent €21 on a present. At the end, she had the fifth of her money. How much money did she have at the beginning?

6 John bought a book, a pencil and a notebook. The book cost the double of the notebook, and the pencil cost the fifth of the book and the notebook together. If he paid €18, what is the price of each article?

Unit 8
Proportions and percentages

8.1 Ratio

A Ratio is a comparison of two or more quantities. Ratios can be used to compare costs, weights, sizes and other quantities.
Each number in a ratio is called a term. You can write ratios in different ways:
Example: 4 parts to 5 parts, 4 to 5, 4 : 5, $\frac{4}{5}\dots$
Equivalent Ratios (or Equal Ratios) are ratios that mean the same.
Example: twenty to forty, is equivalent to five to ten.
A rate is a ratio that expresses how long it takes to do something. To find the rate of a ratio write it as a division.
Example: To walk four kilometers in two hours is to walk at the rate of two km/h.
You can cancel a ratio to its lowest terms, finding the so called simplest form of a ratio. A ratio is normally written using whole numbers only (in its simplest form).
Example: four to five is the simplest form for sixteen to twenty.

8.2 Proportion

Two quantities are in direct proportion if their ratio stays the same as the quantities increase or decrease.
Example: Two pencils cost 7 euro cents. The cost is directly proportional to the number of pencils, so twelve pencils cost 42 euro cents. The rate is the cost of one pencil, €0.035.
A proportion is an equation stating that two ratios are equal. Equal ratios have equal cross products.
Example: $\frac{4}{10} = \frac{6}{15}$ because $4 \cdot 15 = 6 \cdot 10$.
Two quantities are in inverse proportion when one increases at the same rate as the other decreases.
Example: Three men dig a trench in four days. How long would it take six men working at the same rate? 3 multiplied by 2 is 6, 4 divided by 2 is 2. Six men take 2 days.

8.3 Percentages

A percentage (%) is a fraction out of 100.
Example: 30% means $\frac{30}{100} = \frac{3}{10} = 0.3$.
To find a percentage of a quantity change the percentage to a fraction or a decimal and multiply it by the quantity.
Example: 20% of €25 $= \frac{20}{100} \cdot 25 = \frac{500}{100} =$€5.

8.4 Exercises

1 Write down those ratios in the simplest form:

(a) Two to twelve=
(b) Four to twenty=
(c) Five thousand to fifteen=
(d) A hundred and twenty one to forty four=

2 Are those quantities in direct proportion?

(a) Five to twelve and Ten to twenty-four:
(b) Four to six and sixteen to twenty two:
(c) Five to three and fifty five to forty four:
(d) A hundred and twenty one to fifty five and fifty five to twenty five:

3 Six razor blades cost €42. How much will ten razor blades cost?

4 Four packets of tea cost €1.28. How much will three packs cost?

5 Anna buys 12 rubbers for €1.80. How much would it cost her to buy 15 rubbers?

6 Frank is making pastry for 5 apple pies. He always use 4 ounces flour and 2 ounces fat for every pastry. How much of each ingredient does he need?

7 Six people can harvest a field of strawberries in four days. How long would it take eight people to harvest the same field?

8 A field of grass provides enough food for 25 cows for eight days. For how long would the same field feed 10 cows?

9 A factory employs 200 workers. Next year the numbers of workers will decrease by 20% How many workers will be?

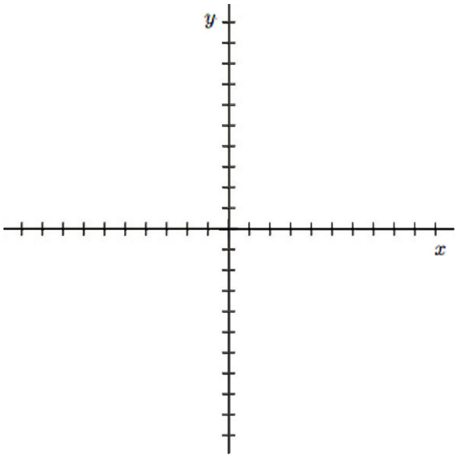
10 A scarf normally cost €20. In a sale there is a reduction of 20%. What is the sale price?

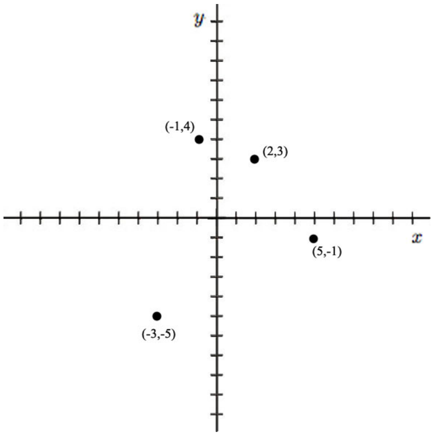
11 The price of a pair of trowser has been reduced by 40% to €30 in a sale. What was the original price?

Unit 9
Functions and graphs

9.1 Cartesian coordinates

To mark a point on a graph you can use Cartesian Coordinates. Cartesian Coordinates marks a point by how far along and how far up it is, so you need to know two values for each point. The horizontal direction is called x and the vertical direction is commonly called y.
An axis is the reference line from which distances are measured. On the x-axis you move to the left and to the right, on the y-axis you move up and down.
To mark a point you need a value x (the first coordinate) for the x-axis and another value y (the second coordinate) for the y-axis. As x increases, the point moves further right. If it decreases, then the point moves further to the left. As y increases, the point moves further up. If it decreases, then the point moves further down.





The point $(0,0)$ is called the origin. The origin is the place where x-axis and y-axis intercept.
Examples: $(2,3)$ means 2 units to the right, and 3 units up. $(-1,4)$ means 1 unit to the left, and 4 units up. $(-3,-5)$ means 3 units to the left, and 5 units down. $(5,-1)$ means 5 units to the right, and 1 unit down.
In a a point, if x is positive and y is positive the point is said to be in quadrant I, if x is negative and y is positive the point is said to be in quadrant II, if x is negative and y is negative the point is said to be in quadrant III, if x is positive and y is negative the point is said to be in quadrant IV.

Examples: $(2,3)$ is in quadrant I. $(-1,4)$ is in quadrant II. $(-3,-5)$ is in quadrant III. $(5,-1)$ is in quadrant IV.

9.2 Function

A function (or a map) is a relation between two sets of quantities (or two variables) in which exactly one element of the first set is paired with each element of the second set. The set of values at which a function is defined is called its domain (the first set) while the set of values that the function can produce is called its range (the second set).
We can define a function by an expression or by an equation.
Example: The function defined by $y = 50x$ can also by defined by the expression "A car travels on a road at a speed of 50 miles per hour", where x is the number of hours and y the miles the car had travelled.

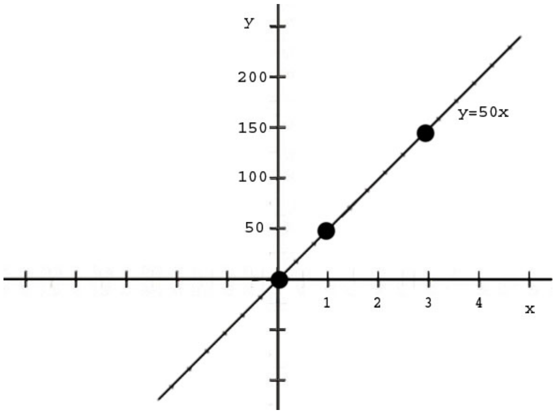
9.3 Graphs and tables of values

Given a function, every value in the domain is uniquely associated with an object in the range. These results can be displayed in a table. A table of values will help you to evaluate and graph the function. Make sure your axes are sensibly labeled with appropriate scales. Don't just start at the first point plotted and end at the last.

Always label your graph fully: the axes and the line with it's equation.
Example:

$f(x) = 50x$

x	0	1	3
y	0	50	150



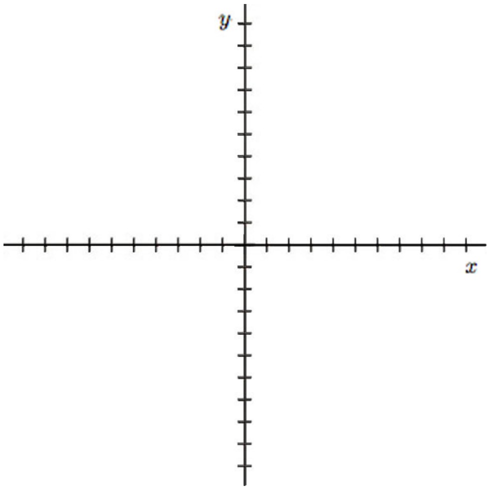
9.4 Exercises

1 Complete:
$(5, 7)$ means 5 units to the ____________ , and 7 units ____________.
$(-4, -7)$ means 4 units to the ____________ , and 7 units ____________.
$(-2, 3)$ means 2 unit to the ____________ , and 3 units ____________.
$(9, -1)$ means 9 units to the ____________ , and 1 unit ____________.

2 Complete:
The point $(-5, 7)$ is in quadrant ____________ .
The point $(5, -7)$ is in quadrant ____________ .

The point $(-5,-7)$ is in quadrant ________________ .
The point $(5,7)$ is in quadrant ________________ .

3 Plot these points in this set of axis:



$A(3,5)$

$B(2,0)$

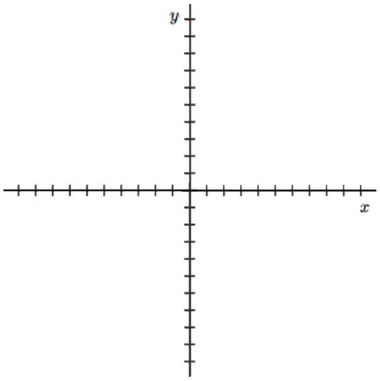
$C(-6,5)$

$D(8,-5)$

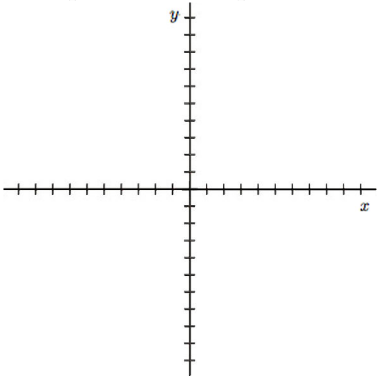
$E(-1,-5)$

$F(0,-5)$

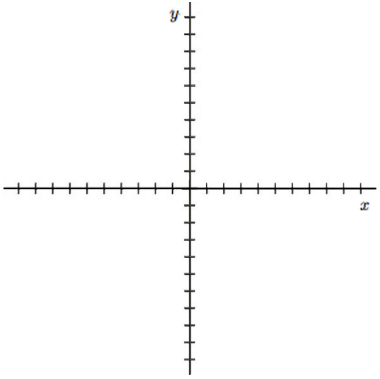
4 Define a function to the expression and draw a graph:
A bus travels on a road at a speed of 20 miles per hour:



The price of a cup of coffee is €1.2 and Mark order two cups a day:



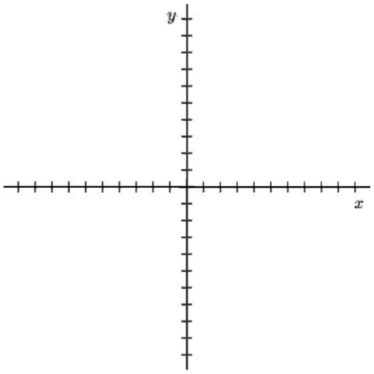
The area of a triangle if the base is 2 cm, and the height is x:



5 Draw a graph for these equations:

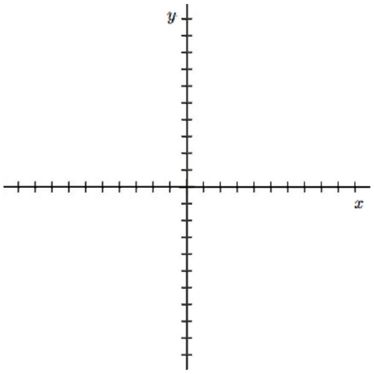
$f(x) = 4$

x			
y			



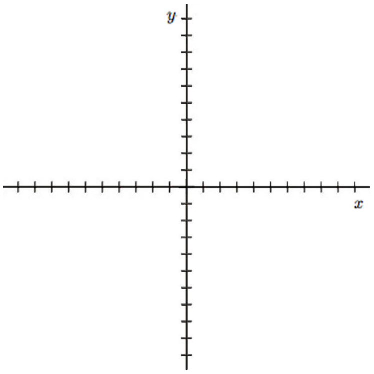
$f(x) = 2x$

x			
y			



$f(x) = 2x + 3$

x			
y			



$f(x) = 1/x$

x							
y